INDIAN STATISTICAL INSTITUTE, CHENNAI CENTRE
Analysis-II

Quiz I **Instructor: S. Ponnusamy**

Date: 26-02-2016 **Time: 10.00-12.00 am** **Total Marks: 20**

Instructions:
- Write your roll number and your name in the answer book
- Use of calculator, mobile phone and mathematical table is not allowed
- Justify your answers by clearly stating the appropriate results/theorems that you use whenever required

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1. Let $\mathbb{Z}[x]$ denote the set of all polynomials with integer coefficients.
$$\mathbb{A} = \{a \in \mathbb{R} : \text{ there exists } P \in \mathbb{Z}[x] \text{ such that } P(a) = 0\}.$$
Is $\mathbb{A}$ closed? Is $\mathbb{A}$ open? Justify your answer. (Hint: $\mathbb{A}$ is countable). **(2 marks)**

2. Determine a largest strip D parallel to the real axis of the z-plane so that e^{-2016z} is a one-to-one mapping of D onto a punctured complex plane **(2 marks)**

3. Determine the largest domain in $\mathbb{C}$ on which $\log(1 - z^3)$ is analytic. **(2 marks)**

4. Let $a_n = n^p$ and $b_n = x^n$, where $p \in \mathbb{R}$ and $x \in \mathbb{R}$. Determine condition(s) in each case so that each of a_n, $a_n/n!$, $b_n/n!$, b_n and $a_n b_n$ approaches 0 as $n \to \infty$. **(4 marks)**

5. Show that the two power series $\sum_{n=1}^{\infty} a_n z^n$ and $\sum_{n=1}^{\infty} n a_n z^n$ have the same radius of convergence. **(4 marks)**

6. Give an example of a power series that converges at every point except 2016 points on the circle of convergence. **(3 marks)**

7. Does the series $\sum_{n=1}^{\infty} \frac{1}{3^{1+\frac{1}{2}+\cdots+\frac{1}{n}}}$ converge? **(3 marks)**

Bonus Mark Questions

8. Let $P(z)$ be a polynomial of degree 2016. Find the radius of convergence of the power series $f(z) = \sum_{n=0}^{\infty} P(n) z^n$.
(Hint: $|P(n)| \to \infty$ as $n \to \infty$). **(4 marks)**

9. Find all the complex numbers for which $\cos z = 2$. **(2 marks)**